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Research Article

Self-Organized Fission-Fusion Control Algorithm for Flocking Systems Based on Intermittent Selective Interaction

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In nature, gregarious animals, insects, or bacteria usually exhibit paradoxical behaviors in the form of group fission and fusion, which exerts an important influence on group's pattern formation, information transfer, and epidemiology. However, the fission-fusion dynamics have received little attention compared to other flocking behavior. In this paper, an intermittent selective interaction based control algorithm for the self-organized fission-fusion behavior of flocking system is proposed, which bridges the gap between the two conflicting behaviors in a unified fashion. Specifically, a hybrid velocity coordination strategy that includes both the egalitarian and selective interactions is proposed, where the egalitarian interaction is to maintain the flock's order and achieve the fusion behavior while the selective interaction strategy is for the response to external stimulus information and generates the fission behavior. Numerical simulations demonstrate that the proposed control algorithm can realize the self-organized fission-fusion behavior of flocking system under a unified framework. The influences of the main control parameters on the performance of the fission-fusion behavior are also discussed. In particular, the trade-off parameter α balances the exploration (fission) and exploitation (fusion) behaviors of flocking system and significantly enhances its movement flexibility and environmental adaptivity.

1. Introduction

In nature, gregarious animals, insects, or bacteria often aggregate into a cohesive group to gain some survival advantages, such as reducing predation risk, improving foraging efficiency, and saving individual energy [1–4]. Within these grouping species, group formation is usually a highly dynamic process: group size and composition may change frequently during the life time of members by group splitting or merging, which is usually referred to as the "fission-fusion" behavior [5, 6] (see Figure 1).

The term "fission-fusion" was firstly introduced by Hans Kummer [7] to describe the social system of a few taxa of nonhuman primates, such as chimpanzees, geladas, and hamadryas baboons, that change the size of their groups by means of the fission and fusion of subunits (called parties or subgroups) [8]. As a matter of fact, fission-fusion behavior is a commonly seen phenomenon in nature. For instance, Bechstein's bats usually aggregate into a large group during pregnancy and lactation for thermoregulation and split into smaller subgroups in the post-lactation period [9]. European starlings often gather into a giant swarm to enhance the possibility of finding food and rapidly segregate into separated subunits in the presence of predator's attack [3]. On the other hand, the fission-fusion behavior is also of practical significance for some artificial flocking systems. For example, a group of unmanned aerial vehicles (UAVs) can gather into a cohesive flock to carpet bomb enemy's targets and split into smaller clusters to avoid the attack of anti-aircraft fire. The autonomous underwater vehicle (AUV) swarm that is executing the seabed exploration task may aggregate into a small group to go across the narrow tunnel and expand to its original state to monitor the mission area. Therefore, investigation on the fission-fusion behavior of flocking system is of both theoretical significant and application value.

Over the last decades, scientists have been working vigorously on the underlying mechanisms of flocking behavior. In [10], a famous flocking model (namely, "Boids"), with three heuristic rules (*cohesion*, *separation,* and *alignment*), was proposed to animate the bird flocking behavior. Later, a simplified discrete-time flocking model for self-propelled

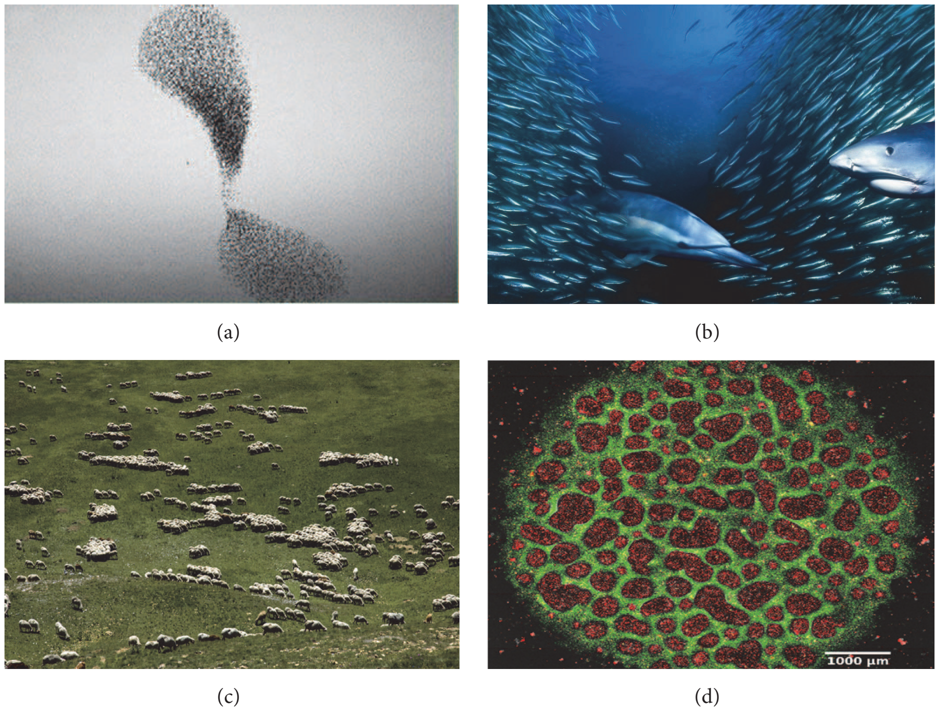

Figure 1: Typical fission-fusion behaviors of flocking system in nature. (a) European starlings, *Sturnus vulgaris*, over Rome [3]; (b) shoal of sardines, in Kwazulu-natal, South Africa; (c) Merinos d'Arles sheep, Ovis aries [6]; (d) segregation in mixed cocultures of primary goldfish keratocytes (PFK, red) and EPC fish keratocytes (EPC, green) [18].

particles considering the *alignment* rule only was introduced in [11]. In addition, a zonal model was proposed in [12], which is able to produce some typical collective behaviors (such as swarm, torus, dynamic parallel, and highly dynamic parallel group) by adjusting certain parameters. In general, these works mainly deploy the egalitarian interaction method and focus on the group fusion aspect; little attention has been paid to the fission behavior. How a randomly distributed flock forms a coherent group and splits into multiple subgroups in a unified manner remains largely unknown.

Recently, owing to the advancements in the high-resolution spatiotemporal flocking data acquisition and analysis techniques, more and more lines of evidence have demonstrated that the selective interaction, rather than the egalitarian interaction, is more effective in the collective response of flocking system [13, 14]. A selective interaction based hierarchical structure was found in pigeon flock's directional choice, which was proved to be more flexible than the egalitarian strategy [15]. In [16], a hybrid control architecture, combining both compromise (egalitarian interaction) and leadership (selective interaction), was proposed to drive the group's movement decisions. A route-dependent switch mechanism between hierarchical and egalitarian strategies in pigeon flocks was found in [17], which revealed that pigeons tend to follow the average of neighbors while moving along a smooth trajectory and select a certain leader to follow when sudden turns or zigzags occurs. Selective interaction, which implements the information transfer by following a specific leader in an implicit manner, shows significant potential in the fission behavior of flocking system.

Inspired by the above empirical evidences, an intermittent selective interaction based control algorithm is proposed for the self-organized fission-fusion behavior of flocking system. The main contributions of this paper are as follows.

(i) A specifically designed intermittent selective interaction behavior is integrated into the conventional average velocity consensus scheme and endows the flock the capability of spontaneous splitting in the presence of external stimulus information.

(ii) A weight adjustment strategy is designed to automatically balance the egalitarian interaction and intermittent selective interaction in different environments, which guarantees the stability and environmental adaptability of flocking system.

(iii) A dynamic threshold value for the fission behavior is designed based on the order parameter of individuals, which make the flock more flexible in the group fission-fusion behavior.

The rest of this paper is organized as follows. In Section 2, the coordinated control problem for the self-organized fission-fusion behavior of flocking system is proposed and the pitfall of the conventional velocity consensus based egalitarian interaction in flocking control is analyzed. In Section 3, a unified fission-fusion control framework is established by introducing the intermittent selective interaction into the egalitarian interaction scheme and a dynamic weight balance strategy is designed for the group fusion and fission behavior. Numerical simulations are provided in Section 4 to verify the effectiveness of the proposed control algorithm and some concluding remarks and future work are drawn in Section 5.

2. Problem Formulation

Consider a flocking system consisting of N identical members; each individual is governed by the following double integrator dynamics:

$$\begin{aligned} \dot{x}_i &= v_i \\ \dot{v}_i &= u_i \end{aligned} \tag{1}$$

where $x_i \in R^n$ is the position vector of individual i, $v_i \in R^n$ denotes its velocity vector, and $u_i \in R^n$ represents the acceleration vector (control input) acting on it.

Constrained by the limited sensing ability, each member can only communicate with its nearby neighbors within a specific range [20]. Hence, the neighboring set of individual i can be denoted by

$$N_i(t) = \{ j : d_{ij} \leq R, \ j = 1, 2, \cdots, N, \ j \neq i \} \tag{2}$$

where $d_{ij} = \| x_i - x_j \|$ denotes the Euclidean distance between individual i and j, and R is the sensing radius of each individual.

Generally speaking, the collective behavior of flocking system is generated by both the local interactions with its nearby neighbors and the external environment [1]. Consequently, the control framework for the self-organized fission-fusion behavior can be roughly formulated as

$$u_i = u_i^{\text{in}} \left(\sum_{j \in N_i(t)} u\left(x_i, x_j, v_i, v_j\right) \right) + g_i u_i^{\text{out}} \tag{3}$$

where u_i^{in} denotes the internal force exerting on individual i from its nearby neighbors, $u(x_i, x_j, v_i, v_j)$ is a specifically designed function for the interaction force between individual i and j, and u_i^{out} is the external force from the environment. In addition, g_i is deployed to show whether individual i is influenced by external information. If $g_i = 1$, we say that the motion of individual i is governed by both its nearby neighbors and surrounding environment; otherwise, $g_i = 0$, and it is only influenced by its neighbors.

Remark 1. Owing to the limited sensing ability, only a small portion of individuals (e.g., that lie on the edge of the flock) can directly sense the external stimuli and are influenced by the external force u_i^{out}, while the motion of others is merely governed by the internal force u_i^{in} from their nearby neighbors [13]. Therefore, the internal force (also called *the local interaction rule*) between individuals plays a key role in the self-organized fission-fusion dynamics of flocking system.

According to the existing literature, most of the internal interactions between individuals follow the *cohesion*, *alignment*, and *separation* rules [19], which can roughly be implemented via the following position and velocity coordination term

$$u_i^{\text{in}} = u_i^{\text{pos}} + u_i^{\text{vel}} \tag{4}$$

Here, the position coordination term u_i^{pos} usually deploys an artificial potential function with long distance attraction and short distance repulsion properties, i.e.,

$$u_i^{\text{pos}} = \sum_{j \in N_i(t)} 2 \left(\frac{a}{\| x_i - x_j \|} - \frac{b}{\| x_i - x_j \|^3} \right) \tag{5}$$

where $a, b > 0$ are parameters that determine the strength of the attraction and repulsion force, respectively.

In addition, u_i^{vel} is the velocity alignment term that guarantees the order of flocking system

$$u_i^{\text{vel}} = - \sum_{j \in N_i(t)} \left(v_i - v_j \right) \tag{6}$$

Equation (6) is the famous "*velocity consensus*" algorithm, which is widely used in the control protocol design of flocking behavior for its mathematical elegance and simplicity [21]. However, such information consensus property (also called *the egalitarian strategy*), although very effective in the fusion behavior, gives a flock the tendency of group cohesion and hampers the process of splitting [22], which may degrade its movement flexibility and reaction rapidity, especially in the presence of external stimulus such as multiple food source or predator's threat [16].

Motivated by this fact, we are trying to develop a novel control framework for the self-organized fission-fusion behavior of flocking system, which aims to realize the spontaneous fusion behavior in free space and the reactive fission behavior under external stimuli in a unified fashion. In particular, this framework is the supplement of the traditional egalitarian strategy, which can deal with the external stimulus in a more flexible and efficient way.

3. Self-Organized Fission-Fusion Control Algorithm Based on Intermittent Selective Interaction

In this section, a unified control algorithm for the self-organized fission-fusion behavior of flocking system is proposed by introducing an intermittent selective interaction into the traditional egalitarian interaction framework, which promotes the stimulus information transfer within the flock and makes it more sensitive to environmental variation.

3.1. Intermittent Selective Interaction Based Control Framework for the Self-Organized Fission-Fusion Behavior. The self-organized fission-fusion behavior of flocking system is virtually composed of two competing parts, where the fusion behavior requires members to form a highly coherent and ordered group, while the fission behavior needs to break up the coherence and split into multiple smaller subgroups [6, 8].

In order to represent the two seemingly contradictory behaviors in a unified manner, an intermittent selective interaction is integrated into the velocity coordination term. Together with the traditional egalitarian interaction based velocity coordination approach, the control framework for the self-organized fission-fusion behavior of flocking system can be generalized as

$$u_i = u_i^{pos} + \underbrace{\alpha u_i^{ega} + (1-\alpha)\, u_i^{sel}}_{u_i^{vel}} + g_i u_i^{out} \quad (7)$$

It is obvious that the control input u_i is composed of three parts.

(1) The position coordination term u_i^{pos}, with the characteristics of long distance attraction and short distance repulsion, follows (5) to coordinate the spatial distribution of individuals.

(2) The velocity coordination term u_i^{vel} is to regulate the alignment of individuals. Here, a hybrid velocity alignment strategy, which combines both the egalitarian and selective interaction based velocity coordination, is proposed via a weighted parameter α:

$$u_i^{vel} = \underbrace{-\alpha \sum_{j\in N_i(t)} \left(v_i - v_j\right)}_{u_i^{ega}} - (1-\alpha) \underbrace{\left(v_i - v_{l_i}\right)}_{u_i^{sel}} \quad (8)$$

where v_{l_i} is the velocity of the temporary leader that individual i selects, and $\alpha \in [0,1]$ is the parameter adjusting the weight between u_i^{ega} and u_i^{sel}.

(3) The external stimuli u_i^{out} have a variety of forms, such as the attraction to a known food source or repulsion to an obstacle or predator's threat [23] (the detailed form of u_i^{out} will be given in Section 4).

Remark 2. In (8), the egalitarian interaction based velocity coordination term u_i^{ega} drives individuals towards their average velocity to form a highly ordered and cohesive flock. On the contrary, the selective interaction based velocity coordination term u_i^{sel} makes it follow the velocity of the temporary leader, which thus endows individual i the potential of splitting from the flock. Specifically, by adjusting the weighting parameter α, the flock can exhibit the self-organized fusion-fission behavior according to the environmental variation adaptively.

Remark 3. It is also worth noting that in (8), the temporary leader of individual i is time varying and evolves according to some *leader selection rule*, which is fundamentally different from the traditional leader-follower approach as our method is to promote the external stimulus information transfer within the flock in an implicit manner.

In the following, we will investigate the temporary leader selection rule from the perspective of promoting external stimulus information transfer within the flock.

3.2. Selective Interaction Based Temporary Leader-Follower Relationship. As has been discussed in Section 2, the self-organized fission-fusion behavior of flocking system is the direct (by directly sensing the external stimulus information) or indirect (through collective propagation of changes in behaviors by group members) response to external stimuli, in which the information flow between individuals plays a significant role [24].

In the free motion of flocking system, the information flow is mainly conducted by averaging all the nearby neighbors' information [25]. However, it is suggested to be more efficient to switch to the single neighbor interaction mode when responding to some external stimuli with abrupt acceleration or sudden turning [17]. Inspired by this fact, a selective interaction rule is proposed from the perspective of maximizing the external information transfer within the flock

$$l_i = \left\{\max C_{ij},\ C_{ij} > C^*,\ j \in N_i(t)\right\} \quad (9)$$

where C_{ij} denotes the influence of neighbor j on individual i, and C^* is the threshold value for the selective interaction.

Remark 4. It can be observed form (9) that individual i will select the most influential neighbor l_i as the temporary leader, which is a widely seen phenomenon in nature as individuals tend to follow the aged, experienced, or strong neighbors to enhance the opportunity of survival [5, 8].

In the fission behavior of flocking system, individuals are required to select the proper temporary leader to promote the stimulus information transfer more efficiently. From the biological visual perception mechanism [24], it is known that individuals are usually very sensitive to the rapid variation of nearby neighbors. On the other hand, members that are moving with fast maneuvering often carry more external stimulus information [14]. Hence, the influence of neighbor j on individual i can be described as

$$C_{ij} = \underbrace{\zeta_{ij} \frac{1}{\left\|x_i - x_j\right\|}}_{A_1} \bullet \underbrace{\kappa_{ij} \frac{\left(v_i \bullet v_j\right)}{\left\|v_i\right\| \left\|v_j\right\|}}_{A_2} \quad (10)$$

where A_1 and A_2 are the influence of neighbor j on individual i with respect to position and velocity, and ζ_{ij} and κ_{ij} are the coefficients of the position and velocity related influence, respectively.

Additionally, a threshold value of C^* is designed to prevent the unexpected fission behavior such as stochastic disordered splitting

$$C^* = e^{-\beta\varphi_i} \quad (11)$$

where $\beta > 0$ is the threshold value adjustment parameter, and φ_i is the order parameter of individual i

$$\varphi_i = \frac{1}{N_i + 1} \left\| \sum_{j=1}^{N_i} \frac{v_j}{\left\|v_j\right\|} \right\| \quad (12)$$

with N_i being the number of nearby neighbors.

Remark 5. It can be seen in (11) that the specifically designed threshold value C^* is a monotone decreasing function with respect to the order parameter φ_i. When $\varphi_i \longrightarrow 0$, the flock moves in a disordered state; the threshold value C^* is high and the selective interaction behavior is not likely to happen, and the flock tends to perform the fusion behavior; when $\varphi_i \longrightarrow$

1, the flock is in an ordered state and the threshold value C^* is small and the selective interaction behavior is easy to occur, which will facilitate the external stimulus information transfer and induce the fission behavior.

4. Simulation Studies

In this section, various numerical demonstrations are provided to verify the feasibility and effectiveness of the proposed fission-fusion control algorithm.

4.1. Performance Metrics. In order to evaluate the performance of the proposed control algorithm in the self-organized fission-fusion behavior of flocking system, a series of metrics are firstly defined as follows.

(1) Polarization Index (ψ). The polarization index ψ represents the velocity alignment degree of all the individuals in the flock

$$\psi = \frac{1}{N}\left\|\sum_{i=1}^{N}\frac{v_i}{\|v_i\|}\right\| \tag{13}$$

where N is the number of all the individuals in the flock, and v_i denotes the velocity of individual i.

It can be seen from (13) that the polarization index varies between $[0,1]$. If the velocities of all the individuals in the flock are aligned, $\psi \approx 1$; otherwise, if the velocity of individuals are completely randomly distributed, $\psi \approx 0$.

Remark 6. It should be noted that the polarization index ψ seems a little like the order parameter φ_i defined in (12). In fact, ψ can be seen as the global order parameter of the whole flocking system, while φ_i only demonstrates the local order parameter of individual i.

Remark 7. The polarization index can well reflect the velocity variation of individuals in the group fusion process. In the group fission process, the velocity of individuals tends to diverge and the polarization index will decline. However, the declining of polarization index is not always corresponding to the fission behavior. Therefore, based on the concept of bimodality coefficient in statistical analysis theory [26], a new performance metric, namely, differentiation index, is introduced.

(2) Differentiation Index (λ). The differentiation index λ demonstrates the velocity divergence degree of individuals in the flock

$$\lambda = \frac{\mathcal{S}^2+1}{\mathcal{K}} \tag{14}$$

where

$$\mathcal{S} = \frac{(1/N)\sum_{i=1}^{N}(v_i-\overline{v})^3}{\sqrt{\left((1/N)\sum_{i=1}^{N}(v_i-\overline{v})^2\right)^3}} \tag{15}$$

$$\mathcal{K} = \frac{(1/N)\sum_{i=1}^{N}(v_i-\overline{v})^4}{\left((1/N)\sum_{i=1}^{N}(v_i-\overline{v})^2\right)^2} - 3 \tag{16}$$

Here, $\mathcal{S}$ and $\mathcal{K}$ denote the skewness and kurtosis of individuals' velocity distribution, respectively.

Remark 8. According to the bimodality coefficient in statistical analysis theory, we can conclude from (14) to (16) that the differentiation index, which reflects the velocity distribution of individuals, also varies between $[0,1]$. To be specific, when $\lambda < 5/9$, the velocity of individuals is in unimodal distribution. Specifically, when $\lambda = 5/9$, the velocity of individuals is in uniform distribution. When $\lambda > 5/9$, the velocity of individuals is in bimodal distribution, which demonstrates that the velocity of individuals tends to diverge. In particular, when $\lambda = 1$, the velocity of individuals is in Bernoulli distribution and the velocity is completely diverged. Therefore, the differentiation index λ is an effective metric to evaluate the fission performance of flocking system.

4.2. Group Fusion and Fission Experiment. Suppose 50 individuals lie randomly within a $15 \times 15\text{m}^2$ region and their initial velocities are all zero. The sensory range of each individual is $R = 5$m. A group formation (fusion) and multitarget tracking (fission) scenario is carried out to show the entire process of group fusion and fission.

Initially, members in the flock aggregate from their initial distributions and move in formation with a consistent velocity $[5\ 5]$m/s under the control protocol (7). Two targets lie on the moving path of the flock symmetrically and their initial positions are $[30\ 35]^T$m and $[35\ 30]^T$m, respectively.

Two members (labelled as individuals 1 and 2) that firstly sense the targets will initiate the fission behavior by tracking the motion of targets towards opposite directions. The tracking force follows

$$u_i^{\text{out}} = -\gamma\left[\left(x_i - x_{\text{tar}_i}\right) + \left(v_i - v_{\text{tar}_i}\right)\right] \tag{17}$$

where x_{tar_i} and v_{tar_i} are, respectively, the position and velocity of target i, and $\gamma > 0$ is the gain of the tracking force.

Two targets, which are also governed by (1), remain stationary until the flock gets into their sensing radius. If the flock approaches the targets, the escaping behavior will be activated, which follows

$$u_{\text{tar}}^{\text{esc}} = c_{\text{tar}}^{\text{esc}}\sum_{j=1}^{S}\exp\left(-\frac{\left\|x_{\text{tar}_k}-x_j\right\|}{l_{\text{esc}}}\right), \quad k=1,2 \tag{18}$$

where $S = \{j : \|x_{\text{tar}_k} - x_j\| \le R_{\text{tar}},\ j = 1,2,\cdots,N\}$ is the set of flock member within the sensing radius of target k, $l_{\text{esc}} > 0$ is the correlation length, and $c_{\text{tar}}^{\text{esc}}$ is the gain of the escaping behavior.

The other simulation parameters are listed in Table 1 and the simulation results are shown in Figures 2 and 3.

Figure 2 is the trajectory of the flocking system in the fusion-fission process. To be specific, Figure 2(a) shows the initial distribution of the flock and targets, where members in the flock lie randomly in the predefined region and the two targets lie on the moving path of the flock symmetrically; Figure 2(b) gives the flock's fusion process, where individuals with random distribution aggregate into a coherent group and organize into a highly ordered

Table 1: Parameters in the simulation.

Parameters	a	b	ζ_{ij}	κ_{ij}	β	α	γ	l_{esc}	R_{tar}	c_{tar}^{esc}
Value	20	3	0.5	2	2.5	0.5	12.3	15m	3m	5.5

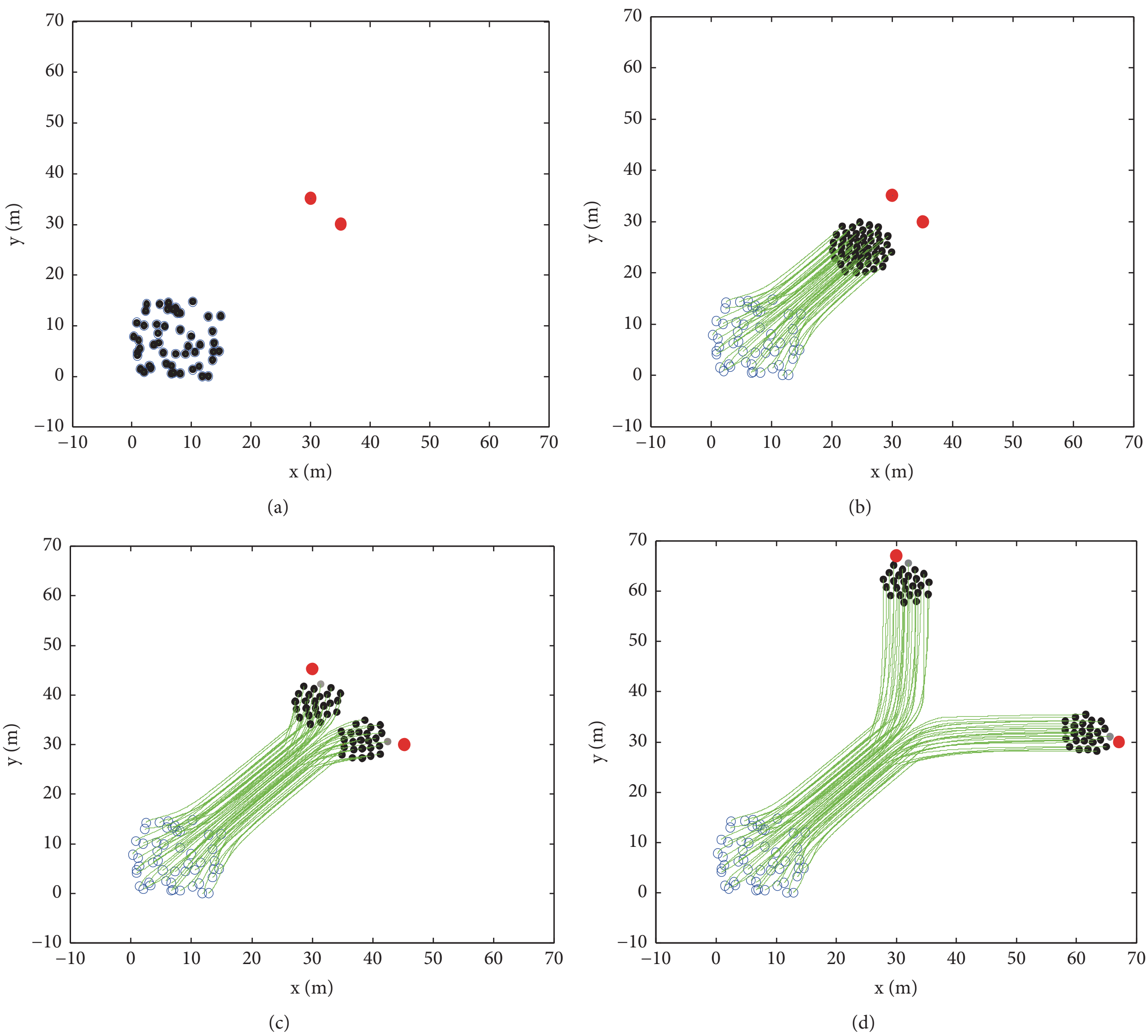

Figure 2: Trajectory of individuals in the fusion-fission process. The blue hollow circles denote the initial positions of individuals, black solid circles demonstrate their final positions, green lines represent the trajectories of individuals, red solid circles are the position of two targets, and gray solid circles are the individuals that firstly sense the targets.

formation to move towards the targets; Figure 2(c) shows the beginning of the fission behavior when the flock encounters two targets; the targets execute the escaping behavior once they detect the flock and the two members that firstly sense the targets try to split form the flock to track the targets separately, and then the flock tends to split into two subgroups under the fission-fusion control algorithm (7). Figure 2(d) illustrates the final position of the flock and the targets, where the coherent flock ultimately splits into two subgroups to track their corresponding targets independently.

Figure 3 demonstrates the velocity variations of individuals during the group fusion-fission process in both X and Y axis. It can be seen that during $t = 0 \sim 4$s the flock is in the fusion phase, and the velocities of all individuals tend to converge to the same value. After that, the fission behavior is activated and the velocity of members tends to diverge, which will ultimately converge to that of the moving targets (the velocities of the two targets are marked in red and black, respectively).

Figure 4 shows the polarization index and differentiation index variation in the group fusion and fission process, from which one can clearly see that, in the group fusion process, the polarization index ψ increases from a very small value to 1 in finite time, which suggests that the flock aggregates from

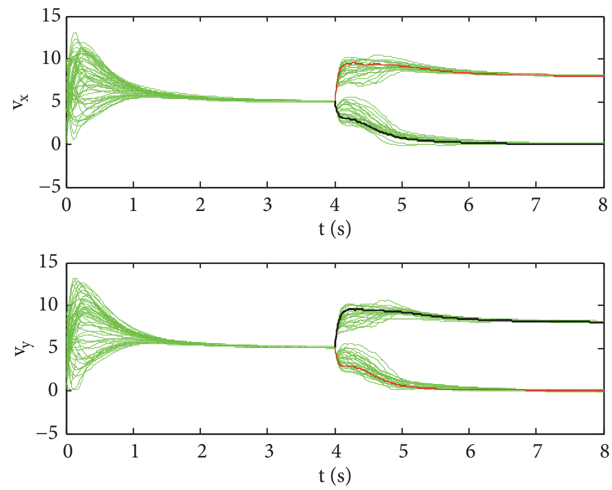

Figure 3: Velocities of individuals in the fusion-fission process.

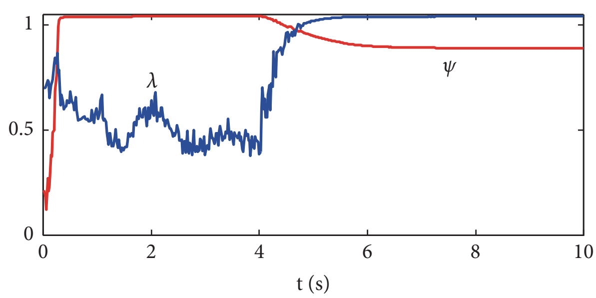

Figure 4: The polarization index and differentiation index in the fusion-fission process.

random distribution to an ordered state and the velocities of individuals are well aligned. The differentiation index λ, on the other hand, fluctuates around 0.5 and the velocity of individuals is in unimodal or uniform distribution. After $t = 4$s, the flock fission process begins, and the polarization index ψ tends to decrease due to the velocity divergence of individuals. The differentiation index λ begins to increase and ultimately converges to 1, which illustrates that the velocity of individuals is in bimodal distribution and the fission behavior completes.

From the above simulation results, one can clearly see that the proposed fission-fusion control algorithm (7) can make the flock aggregate into a coherent group in free space and segregate into multiple subgroups when a small portion of members split from the flock. Therefore, the proposed control algorithm guarantees both the order and flexibility of flocking system, which is much more universal than the traditional flocking control method and also very crucial for the survival and evolution of flocking system.

4.3. Comparison with Conventional Egalitarian Strategy. In order to demonstrate the superiority of the proposed control algorithm (7), a comparative simulation is carried out with a typical consensus based control method proposed in [19], where the velocity of members in the flock is

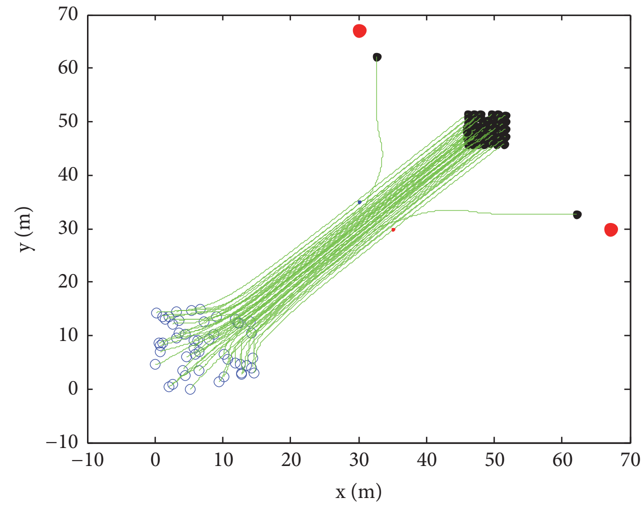

Figure 5: Trajectory of individuals under the control algorithm of [19].

coordinated according to (6). The other settings are the same as Section 4.2.

Figure 5 demonstrates that the consensus based control method can also achieve the fusion behavior, where scattered distributed individuals aggregate into a highly ordered formation. However, this approach is unable to realize the fission behavior when two targets move towards opposite directions. Only the two individuals that sense the targets can split from the flock and track the moving targets; the other members still move along their original path. This is largely due to the inherent coherence property of the consensus based velocity coordination term, which hampers the flock's splitting process and degrades its efficiency in response to external stimuli.

Figure 6 shows the velocity of individuals in both X and Y axis, where green lines are the velocity curves of individuals that are not directly influenced by external stimuli, and red and black lines are the velocity curves of individual 1 and individual 2, respectively. It can be clearly seen that members in the flock firstly aggregate into a coherent group under the consensus based control scheme, and their velocities eventually converge to the same value. However, when external stimuli act on partial members of the flock, only two individuals split from the flock, and the rest of the flock remain unchanged and keep their original movement direction.

Figure 7 shows the polarization index and differentiation index variation under the control algorithm of [19]. In the group fusion process, both the polarization index and differentiation index variation are the same as those of Figure 4, which illustrates that the control algorithm of [19] can well accomplish the fusion behavior. In the group fission process, the polarization index ψ remains about 1 (the whole flock is generally in an ordered state). However, the differentiation index is not significantly increased (fluctuates around 0.55), which implies that the velocity of individuals is still in unimodal distribution and thus the fission behavior does not

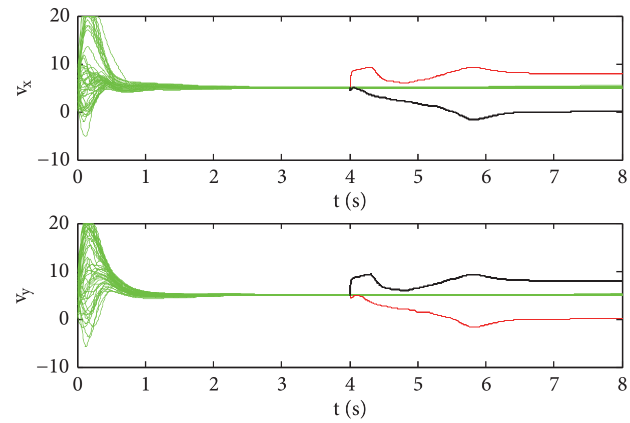

FIGURE 6: Velocity of individuals under the control algorithm of [19].

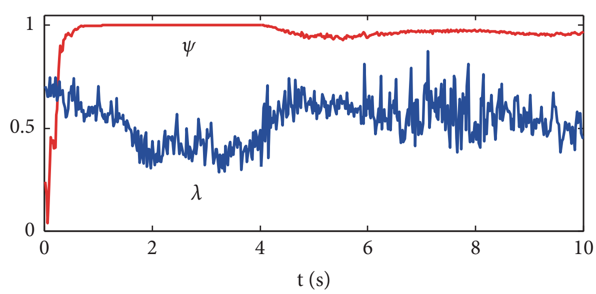

FIGURE 7: The polarization index and differentiation index under the control algorithm of [19].

occur. Therefore, the control algorithm of [19] cannot realize the self-organized fission behavior in the presence of external stimuli.

Compared with the fusion-fission control algorithm proposed in this paper, the traditional velocity consensus based control method hampers the external information transfer within the flock and is unable to realize the self-organized fission behavior when flock encounters some external stimuli. Therefore, our approach is more efficient in promoting local stimulus information transfer and makes the flock more adaptable to the dramatically changing environment.

4.4. The Reunion/Rejion Capability of Flocking System. The simulation in Section 4.2 demonstrates the self-organized fusion and fission capabilities of flocking system. However, whether the separated subgroups can reunite into a single flock still remains unknown. Here, an additional simulation is carried out to verify the reunion capability of the flock.

The two targets are supposed to disappear at $t = 8$s. After that, an additional navigational term is incorporated into algorithm (7) with the following form

$$u_i = u_i^{pos} + \underbrace{\alpha u_i^{ega} + (1-\alpha) u_i^{sel}}_{u_i^{vel}} + u_i^{nav} + g_i u_i^{out} \quad (19)$$

where $u_i^{nav} = -\varrho(v_i - v_{nav}i)$, $i = 1, 2$, with ϱ being the navigational gain and $v_{nav}i$ being the navigational velocity of subgroup i.

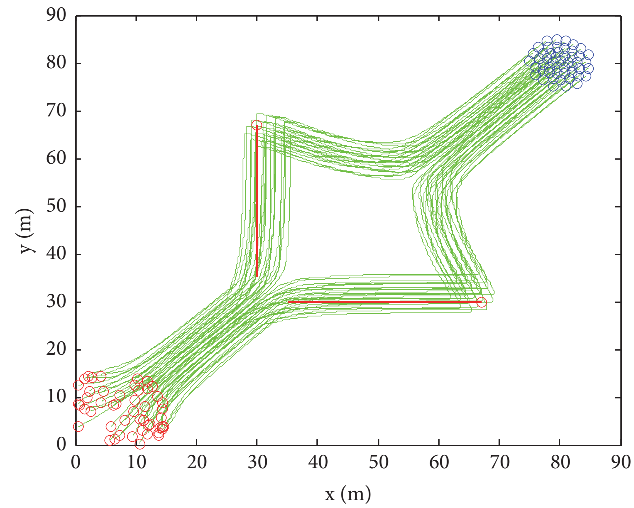

FIGURE 8: The reunion capability of the flocking system after splitting.

Here, we let $\varrho = 10$, $v_{nav}1 = [-5, 5]$m/s, and $v_{nav}2 = [5, -5]$m/s. The navigational term u_i^{nav} drives the two subgroups to move close to each other. Once they are within the sensing range of any individuals in the flock, the navigation term disappears and the flock begins to implement the reunion behavior based on the local interactions only.

Figure 8 shows the reunion behavior of the flocking system after splitting, which suggests that the proposed fission-fusion coordination algorithm (7) can make the separated subgroups reunite into a coherent single flock. This is because when two targets disappear, the external stimuli acting on the flock no longer exist, and the navigational term drives the two subgroups to move close to each other. Once they are within the sensing range of any individuals in the flock, the navigation term disappears and individuals determine their motion based on the local interaction with nearby neighbors only. Meanwhile, there is no abrupt turning or acceleration of any single individual and thus the selective interaction behavior will not occur, which makes the selective interaction based velocity coordination term lose efficacy in the fission-fusion control algorithm (7). Therefore, they will execute the group fusion law and regather into a single flock as long as they are in the sensing range of each other.

4.5. Discussion: The Influence of Parameters on the Fission-Fusion Behavior. As seen in Table 1, there are many parameters in the fission-fusion control algorithm of flocking system, which together determine the performance of fission-fusion behavior. In order to better illustrate the influence of the main parameters on the fission-fusion behavior of flocking system, a detailed discussion is given as follows.

4.5.1. The Influence of Parameters a and b on Flock's Density. The parameters a and b in the artificial potential function (5) is to determine the relative distance between individuals in the steady state. From (5) we know that $d_0 = \sqrt{b/a}$ is the equilibrium distance between the attraction and repulsion

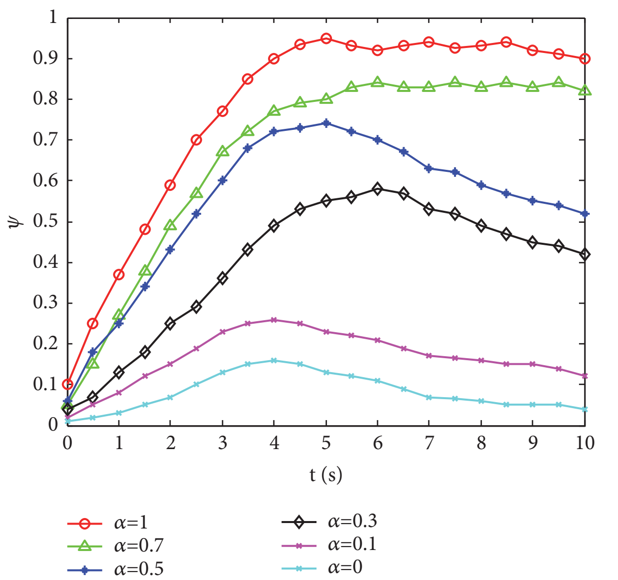

FIGURE 9: The polarization index of flocking system with different α.

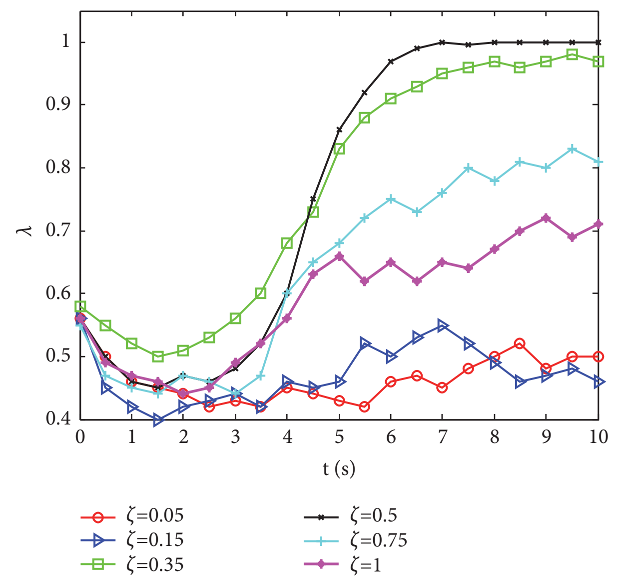

FIGURE 10: The differentiation index of flocking system with different ζ_{ij}.

forces. If $d_{ij} > \sqrt{b/a}$, the attraction force will be dominant, which drives individuals to move towards each other and the flock size will shrink. On the contrary, if $d_{ij} < \sqrt{b/a}$, the repulsion force will be dominate and individuals tend to move away from each other; meanwhile the group size will expand. Therefore, $\sqrt{b/a}$ is relevant to the density of flocking system.

On the other hand, if the equilibrium distance is larger than the sensing range R of each individual, i.e., $\sqrt{b/a} > R$, members in the flock are unable to get the information of neighbors and the fusion behavior cannot be guaranteed. Therefore, the feasible range of parameters a and b should satisfy $\sqrt{b/a} \in (0, R]$.

4.5.2. The Influence of Parameter α on the Polarization Index ψ. It is known that the parameter α in (8) plays a significant role in the fusion-fission control of flocking system, as it balances the weights of egalitarian interaction and selective interaction in the velocity coordination of individuals. Here, an extensive investigation is carried out to demonstrate the influence of parameter α on the polarization index ψ of fission-fusion behavior.

Figure 9 shows the polarization index variation with different α in the flock fission-fusion process, where the flock is expected to execute the fusion behavior from $t = 0 \sim 4$s and perform the fission behavior afterwards. We can see that if α is high (e.g., $\alpha = 1$), the polarization index increases rapidly from $0.1 \sim 0.9$, which shows that the flock aggregates from random distribution to an ordered state and the formation process is accomplished. However, the polarization index still maintains a high value (about 0.9) when the fission behavior begins, suggesting that the flock still moves in a coherent formation and the group fails to achieve the fission behavior. With the decline of α, the polarization index in the fission process decreases synchronously, which shows that the fission behavior tends to happen. If α further decreases, the polarization index will keep a low value in the fusion process and group fusion behavior is not likely to happen. Therefore, a moderate α (about 0.5) can make the flock perform the fusion and fission behavior simultaneously and endows the flock the ability of implementing the fusion behavior in free space and spontaneous fission behavior under external stimuli.

From the above results, it can be concluded that the parameter α realizes the trade-off of exploration (external stimulus information) and exploitation (the existing information in flocks) of flocking system in the group fission and fusion process. As α increases, the "exploitation" behavior is highlighted, and the flocking system tends to use its internal information to aggregate into a coherent group. On the contrary, with the decrease of α, the "exploration" behavior is strengthened and the flock tends to split via the external information. In nature, exploration and exploitation are a very commonly seen phenomenon in flocking system, where exploration enhances the possibility of finding food or decreases the rate of being prey, while exploitation maintains the original motion of flocking system and keeps its stability. Therefore, the method proposed in this paper is consistent with the characteristic of natural flocking system, which is more adaptable than the traditional flocking control method.

4.5.3. The influence of Parameters ζ_{ij} and κ_{ij} on the Differentiation Index λ. The parameters ζ_{ij} and κ_{ij} are the coefficients of the position and velocity related influence in (10), which determine the temporary leader to follow and influence the fission behavior. Here, a comprehensive study is performed to show the influence of parameters ζ_{ij} and κ_{ij} on the differentiation index λ.

From Figure 10 we can see that the differentiation index is in bimodal distribution (the fission behavior emerges) if the coefficient of the position related influence term

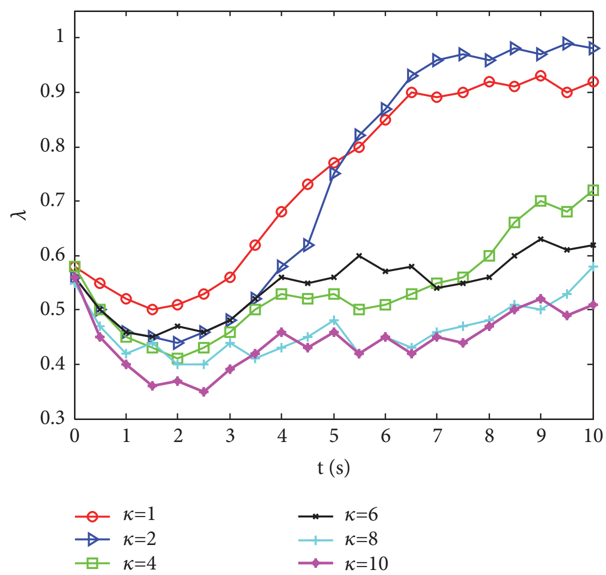

FIGURE 11: The differentiation index of flocking system with different κ_{ij}.

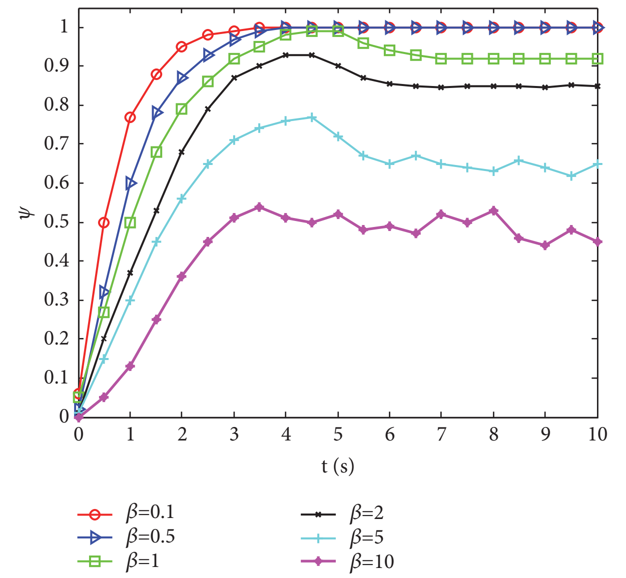

FIGURE 12: The polarization index of flocking system with different β.

$\zeta_{ij} \in [0.35, 1]$. Specifically, the fission behavior is well accomplished if $\zeta_{ij} = 0.5$ (the differentiation index $\lambda \approx 1$ and the velocity of individuals is in Bernoulli distribution). If $\zeta_{ij} < 0.5$ (e.g., $\zeta_{ij} = 0.35$), the fission behavior can also be achieved, although the differentiation index λ is less than 1. However, if ζ_{ij} continues to decline, the differentiation index λ tends to be less than 5/9; the velocity of individuals is in unimodal distribution and the fission behavior does not occur. On the other hand, if $\zeta_{ij} > 0.5$, we can see that $\lambda > 5/9$. Although the velocities of individuals are diverged, it may not guarantee a successful fission behavior because λ is much less than 1.

Based on the above discussion, we can conclude that relatively moderate position correlation is beneficial for the fission behavior of flocking system. Too large position correlation will make individuals follow the motion of its nearest neighbor, which may neglect the stimulus information hidden in the neighborhood and cause the failure of fission behavior. However, if the position correlation is too small, the fission behavior may also fail because the flock may be in a disordered state without considering the position distribution of nearby neighbors.

In Figure 11, it is observed that the fission behavior is very sensitive to the variation of the coefficient of the velocity related influence term κ_{ij}. If κ_{ij} is small (e.g., $\kappa_{ij} \leq 2$), the fission behavior can be accomplished. However, with the increase of κ_{ij}, the differentiation index λ declines dramatically, which cannot achieve the fission behavior of flocking system. The main reason lies in that too strong velocity correlation may make individuals too sensitive to the velocity variation of nearby neighbors, and hence the order and stability of the flocking system cannot be guaranteed.

Therefore, a relatively small κ_{ij} can make individuals obtain appropriate velocity correlation with nearby neighbors, meanwhile maintaining the order of the flock.

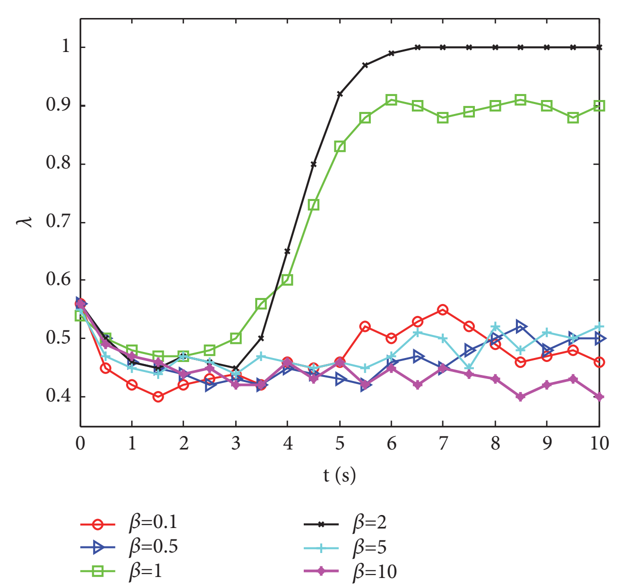

FIGURE 13: The differentiation index of flocking system with different β.

4.5.4. The Influence of Parameter β on the Polarization Index ψ and Differentiation Index λ. As is shown in (11), the threshold value C^* for the fission behavior is time varying along with the variation of the order parameter φ_i, where the adjustment parameter β is to determine the steepness of the threshold value curve. In the following, numerical simulations are performed to show the influence of parameter β on the polarization index ψ and differentiation index λ.

Figures 12 and 13 show the polarization index and differentiation index variation with different β. It can be clearly observed that when β is small (e.g., $\beta < 1$), the polarization index ψ increases from 0 to 1 rapidly, which well accomplishes the fusion behavior. Meanwhile, the differentiation index λ fluctuates around 0.5, suggesting that the velocity of individuals is in unimodal distribution. However, the polarization index and differentiation index still keep unaltered in the group fission process, which illustrates the failure of fission behavior. Therefore, too small β is unable to realize the fission behavior. The main reason lies in that when β is small, threshold value curve is complanate and C^* maintains a high value, which prevents the occurrence of the fission behavior.

From Figures 12 and 13, we can also see that with the increase of β $(1 < \beta < 2)$, the group fusion behavior can be achieved (i.e., $\psi \approx 1,\ \lambda \approx 0.5$). In the group fission process ($t > 4$s), the polarization index ψ tends to decline moderately, while the differentiation index λ begins to increase ($\lambda > 5/9$), which implies that the velocities of individuals are in bimodal distribution and hence the fission behavior occurs.

Moreover, if β continues to increase, the threshold value curve becomes steep and C^* tends to decline. Thus, the fission behavior becomes easy to occur and the flock tends to split in a stochastic fashion. Therefore, the order of the flock is not well maintained and both the fusion and fission behavior cannot be accomplished in an ordered way.

5. Conclusions and Future Work

This paper investigates the self-organized fission-fusion control problem of flocking system. A hybrid velocity regulation mechanism, which combines the selective interaction based velocity coordination into the traditional average consensus rule, is proposed to endow a flock the capability of aggregating in free space and spontaneous splitting in the presence of external stimulus. Various numerical simulations demonstrate the feasibility and effectiveness of the proposed method in flocking fusion-fission control. This paper bridges the gap between flock fission and fusion behavior, which is expected to provide new insight into the coordinated control of flocking system.

In this paper, there are many parameters in the self-organized fission-fusion control algorithm of flocking system and we conduct the parameter selection procedure in a trial-error method. Although the influences of the main parameters on the performance of the fission-fusion behavior are discussed in detail, how to select the appropriate parameter set in an automatic and efficient way remains largely unknown. In the future, we will investigate the control parameter selection method via some optimization techniques, such as the genetic algorithm.

Data Availability

The data used to support the findings of this study have not been made available because another paper is in preparation based on these data.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

Acknowledgments

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